ROLL NO....................

MATH. 203/22

II SEMESTER EXAMINATION, 2022

M. Sc. (MATHEMATICS)

PAPER-III

GENERAL & ALGEBRAIC TOPOLOGY

TIME: 3 HOURS **MAX.- 80**
MIN.- 16

Note: The question paper consists of three sections A, B & C. All questions are compulsory.
Section A- Attempt all multiple choice questions.
Section B- Attempt one question from each unit.
Section C- Attempt one question from each unit.

SECTION 'A'
MCQ (Multiple choice questions)

$2 \times 8 = 16$

1. Which is not true?

(a) Product of non-empty class of compact spaces is compact

(b) Product of non-empty class of first countable spaces is first countable.

(c) Product of non-empty class of connected spaces is connected

(d) Product of non-empty class of path connected spaces is path connected.

2. If $X = \prod_{i \in I} X_i$, then which is true for $B_i \subseteq X_i$:

(a) $\prod_{i \in I} B_i$ is a wall

(b) $\prod_i^{-i}(B_i)$ is a box

(c) $\prod_i(B_i)$ is a wall

(d) $\prod_{i \in I} B_i$ is a box

3. An indexed family $\{f_i : i \in I\}$ of functions on X distinguishes points if for $x \neq y$ in X, there exists $j \in I$ such that :

(a) $f_i(x) = f_j(y)$ (b) $\pi_{jo} f_j(x) \neq \pi_{jo} f_j(y)$

(c) $f_j(x) \neq f_j(y)$ (d) $\pi_{jo} f_j(x) = \pi_{jo} f_j(y)$

4. A Hilbert cube is a space of the form -

(a) $(0,1)^I$ (b) $[-1,1]^I$

(c) $[0,1]^I$ (d) $[-1,1]^I$

5. Which is not true?

(a) Set N is directed by relation $'' \geq ''$ in usual sense.

(b) Set Q is directed by relation $'' \geq ''$ in usual sense.

(c) The collection of all neighbourhoods of points in topological space is directed by inclusion relation.

(d) Set R is directed by relation $'' \geq ''$ in usual sense.

6. Which is not a property of filter F in X?

(a) F is closed under finite intersection.

(b) Power set P(X) is finite on X

(c) F has finite intersection property

(d) X is an elements of F.

Q.2. State and prove Tychonoff embedding theorem.

OR

State and prove Urysohn metrization theorem.

Q.3. A point $x_0 \in X$ is a cluster point of the net S if and only if there exists a subnet T which converges to x_0.

OR

Let X, Y be a topological spaces, $x \in X$ and $f: X \to Y$ a function. Then f is continuous at x if and only if whenever a filter F converges to x in X, the image $f(F)$ converges to $f(x) in$ Y.

Q.4. Show that the fundamental group of circle is infinite cyclic.

OR

Define homotopy of paths. Show that the path homotopy is an equivalence relation.

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7. Every open covering of a regular space X has a refinement then which statement is not equivalent to others?
   (a) An open covering of X and countably locally finite
   (b) An open covering of X and locally finite.
   (c) A closed covering of X and countably locally finite.
   (d) A closed covering of X and locally finite.

8. Which is not true?
   (a) Every paracompact space is normal
   (b) Every metrizable space is paracompact
   (c) Every compact separable metrizable Hausdorff space is first countable
   (d) Every regular Linde loff space is paracompact

SECTION 'B' $4 \times 6 = 24$

Short Answer Type Questions (Word limit 200-250 words.)

Q.1. State and prove Tychonoff theorem.

OR

The product topology is the Coarser topology for which projection functions are continuous.

Q.2. If the product is non-empty then each coordinate space is embeddable in it.

OR

Show that every paracompact space is normal.

Q.3. Let $S: D \to X$ be a net in a topological space and let $x \in X$. Then x is a cluster point of S if and only if there exists a subnet of S which converges to x in X.

OR

Let C be a non-empty family of subsets of a non-empty set X. Then there exists a filter on X containing C if and only if C has the FIP.

Q.4. Define covering space. Show that a covering map is a local homeomorphism.

OR

If X is locally connected then a continuous mapping $P: \bar{X} \to X$ is covering map if and only if for each component H of X , the mapping $P: P^{-1}(H) \to H$ is a covering map.

SECTION 'C' $4 \times 10 = 40$

Long Answer questions (Word limit 400-450 words.)

Q.1. State and prove Generalised Heine Borel theorem.

OR

A product space is locally connected if and only if each coordinate space is locally connected and all except finitely many of them are connected.